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THE CONTRIBUTION OF TWIST-4 TO THE  $Q^2$  EVOLUTION OF  
 $F_2$  AND  $xF_3$  - AN EXPERIMENTAL REVIEW\*

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The Contribution of Twist-4 to the  $Q^2$  Evolution of  
 $F_2$  and  $xF_3$  - An Experimental Review\*

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ABSTRACT

The status of the theoretical and experimental study of higher twist contributions to the nucleon structure functions is reviewed. After noting the dangers of combining experiments with widely different  $\langle Q^2 \rangle$  and targets, emphasis is placed on those results coming from a single experiment. The values of  $\Lambda$ , the twist-2 scale factor, and  $h_4$ , the coefficient of  $x/Q^2(1-x)$ , are restricted by:  $\Lambda < 0.44 \text{ GeV}$  and  $-0.2 < h_4 < 0.5 \text{ GeV}^2$ .

The precision with which the nucleon structure functions -  $F_2$  and  $xF_3$  - can be measured has increased significantly in the past few years. Improved detectors and refined experimental techniques have allowed a relatively accurate determination of these functions over an expanding  $x$ - $Q^2$  domain. There is no longer any doubt that scale breaking has been observed, i.e.,

$$F_i(x) \rightarrow F_i(x, Q^2),$$

and the emphasis is being placed on comprehending the processes which conspire to yield scale breaking.

The present understanding of the subject indicates that the processes can be divided into two general categories; those characterized by the kinematics of the interaction and those effects dependent on the dynamics of the interaction. After briefly describing the method used to account for the kinematic  $Q^2$  dependence, the balance of this review will concentrate on the dynamic processes (QCD) with emphasis on the non-perturbative, higher twist contribution.

#### KINEMATIC - Target Mass Effects

The kinematical effects of target and constituent masses can be accounted for by using the variable  $\xi$  instead of  $x$  where

$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2 x^2 / Q^2}} .$$

This variable was introduced by Georgi and Politzer<sup>1</sup> in 1976 and, although it may not prove to be the definitive<sup>2</sup> method for making this correction, it has been used by all experiments summarized in this review. Figure 1 shows how a  $Q^2$  independent function would be distorted by these target mass effects and appear to have a  $Q^2$  dependence. Thus, if all  $Q^2$  dependence were due to kinematics, a simple rescaling of  $F(x, Q^2) \rightarrow F(\xi)$  should remove all  $Q^2$  variations in the structure functions. This is not the case.<sup>3,7,17</sup>

#### DYNAMIC - Perturbative Quantum Chromodynamics (Twist-2)

The dynamical effects, which influence the  $Q^2$  evolution of the structure functions, are predicted in the context of the theory of strong interactions - QCD. Up to now, it has been possible to develop a perturbative expansion only for the leading twist (twist-2) term. Many experiments have attempted to extract the value of  $\Lambda$ , the twist-2 scale parameter, from the measurement of  $F_1(x, Q^2)$ . There are two main ways of doing this; using the moments of the structure functions

$$M_N^i(Q^2) = \int_0^1 x^N F_i(x, Q^2) dx$$

or direct examination of the  $F_i(x, Q^2)$ . The moment method is subject to several severe disadvantages. For each  $Q^2$  region there is an  $x_{\min}$  below which there is no data. Not only does one have to extrapolate over this low  $x$  region ( $0 < x < x_{\min}$ ), but also the high  $x$  region, which becomes increasingly important for higher moments, is subject to large and poorly known smearing corrections (Fermi motion). The size and uncertainty of the Fermi motion smearing factor is shown in Fig. 2. It demonstrates that at low  $Q^2$  and high  $x$  the smearing factor is not known to better than (25-50)%. On the other hand, in the direct analysis of the structure functions, the theoretical predictions can be tested directly within the  $x$  limits dictated by the experiment.

The overall  $Q^2$  behavior of the structure functions and moments is predicted to be of the form  $\sim \ln^{-1}(Q^2/\Lambda^2)$  in lowest order and  $\ln[\ln(Q^2/\Lambda^2)] \ln^2(Q^2/\Lambda^2)$  in the second order of the perturbative twist-2 expansion. The agreement between the various experiments is good (not excellent) and the average values of  $\Lambda$  are

$$\langle \Lambda_{LO} \rangle \approx 0.18 \pm 0.7 \text{ GeV},$$

$$\langle \Lambda_{\overline{MS}} \rangle \approx 0.16 \pm 0.5 \text{ GeV},$$

where  $\overline{MS}$  refers to the renormalization scheme used in the second order term of the expansion.

Is this sufficient? Can target mass corrections and twist-2 QCD account for all the observed  $Q^2$  dependence of the structure functions. If there is no further  $Q^2$  dependent contributions to the structure functions, then  $\Lambda$  extracted from  $F_1$ , corrected for target mass effects, should be independent of  $x$ ,  $Q^2$  and  $W$ . This point has recently been addressed<sup>4,5</sup> using the data of the SLAC-MIT<sup>6</sup> (electroproductions), CDHS<sup>7</sup> ( $\nu, \bar{\nu}$  scattering) and EMC<sup>8</sup> (muoproduction) collaborations. Figure 3 shows the results of their analysis. It indicates that there is still an  $x$  and  $W^2$  dependence of  $\Lambda$  in both lowest and second order expressions, and it thus appears that there are further processes which contribute to scale breaking.

#### DYNAMIC - Higher Twist Contributions

The higher twist terms are thought to account for phenomena important at low  $Q^2$  such as di-quark scattering, resonant final states and the parton  $p_T$ . The  $Q^2$  fall-off of the higher twist terms is more rapid compared to the  $\sim 1/\ln(Q^2)$  of the leading twist expression.

As a first observation it should be noted that these higher twist terms are extremely difficult to study, both experimentally and theoretically. Experimentally, where the higher twist contributions are expected to be largest, statistics are poorest and smearing corrections are most uncertain. Theoretically a new dimension is introduced into the expressions for (i.e.) the moments of the structure functions

$$M_N = \sum_{\substack{t=2 \\ \text{even}}} \frac{A_N^t}{(Q^2)^{t-2}} \left[ \alpha(Q^2) \right]^{d_N^t} \left[ 1 + R_N^t \frac{\alpha(Q^2)}{\pi} + \text{---} \right]. \quad (1)$$

There are many operators of twist  $\geq 4$  which contribute to this expression, and the anomalous dimensions  $(d_N^t)$ , which have been calculated in twist-2, have been calculated<sup>9</sup> for only a few of the twist-4 operators. The many  $A_N^4$  which contribute to the twist-4 term can only be estimated by assuming a certain model. These  $A_N^4$  should, in principle, be extracted experimentally once the other parameters can be theoretically calculated.

For lack of a rigorous theoretical derivation, the higher twist terms (twist-4, twist-6 ---) are assumed to act as power corrections to the perturbative twist-2 contribution. They introduce terms such as

$$M_N(Q^2) = M_N^{t=2} \left[ 1 + \frac{M_4^2}{Q^2} + \frac{M_6^4}{Q^4} + \dots \right] \quad (2)$$

to the moments and

$$F_i(x, Q^2) = F_i^{t=2}(x, Q^2) \left[ 1 + \frac{\mu_4(x)}{Q^2} + \frac{\mu_6(x)}{Q^4} + \dots \right] \quad (3)$$

to the structure functions.

There have been several attempts to estimate the  $M_4^2$  term of expression 2. They all involve the simultaneous determination of  $M_4^2$  and the twist-2 parameter  $\Lambda$ . De Rújula, Georgi and Politzer<sup>10</sup> used arguments based on duality for the  $N=2$  moment and found  $M_4^2 \approx 0.14 \text{ GeV}^2$  with  $\Lambda = 0.5 \pm 0.2$  while Glück and Reya<sup>11</sup> inserted the twist-2 plus twist-4 expressions for the  $N=2$  moment in the Altarelli-Parisi<sup>12</sup> evolution equation and found  $M_4^2 \leq 0.03 \text{ GeV}^2$ ,  $\Lambda = 0.5 \text{ GeV}$ . The value of  $\Lambda (= .5 \text{ GeV})$  from these two analyses is quite large. A very ambitious calculation by Jaffe and Soldate<sup>13</sup> used the operator product expansion plus the MIT bag model and found  $M_4^2 \leq .01 \text{ GeV}^2$ ,  $\Lambda \approx .25 \text{ GeV}$  with the sign of the twist-4 term determined to be negative.

There are other important theoretical contributions which have not yielded numerical estimates. Among these is the diagrammatic approach suggested by Politzer<sup>14</sup> and attempts<sup>15,16</sup> to recast the operator product expansion results in terms of the more intuitive parton model. In looking at this brief summary of the theoretical efforts to



date, it should be noted that theoretical speculation has been limited to the  $N=2$  moment of the structure functions. All higher moments and, in particular, the twist-4 contribution to the structure functions themselves have yet to be studied.

Experimentally, as in the theory, the examination of higher twist contributions is just beginning. Two collaborations have tried to extract the twist-4 term from the moments of  $x F_3(x, Q^2)$ . The CDHS group combined their data<sup>7</sup> with the low  $Q^2$  SLAC<sup>6</sup> results and examined  $M_2(Q^2)$ . The allowed range from experimental and theoretical uncertainties is

$$-.2 < M_4^2 < .2 \text{ GeV}^2, \Lambda_{LO} < 0.6 \text{ GeV} \quad .$$

The BEBC results<sup>17</sup> were combined with the low energy (PS) Gargamelle results and the first, third and fifth moments were used to try to extract  $M_4^2$ . The results for the  $N=1$  moment, the Gross-Llewellyn Smith sum-rule, have been published<sup>18</sup> separately. Figure 4 shows the results where the dashed line represents the limit at which  $\alpha_s/\pi$  (or  $|M_4^2|/Q^2$ )  $> 1.0$ . A safe limit would then be

$$|M_4^2| < .12 \text{ GeV}^2, \Lambda < .35 \text{ GeV} \quad .$$

The results for the  $N=3$  and  $N=5$  moments can be found in Fig. 5: As can be seen there are strong correlations between  $M_4$  and  $\Lambda$ . To summarize these few results from the moment analysis, the favored solutions lie within the bounds

$$-.1 < M_4^2 < .2 \text{ GeV}^2 \quad \text{and} \quad \Lambda \leq .3 \text{ GeV} .$$

In turning to an analysis of the structure functions themselves, although the experimental values are more reliable than the moments, there is one extra unknown - the  $x$  dependence of the twist-4 contribution. There have been only two attempts to experimentally extract this  $x$  dependence. Combining EMC and SLAC data, an  $x$  dependence of  $x^2/(1-x)^2$  is strongly favored (see Fig. 6). While combining CDHS + SLAC and fitting to the form

$$F_2 = F_2^{\text{twist-2}} \left[ 1 - \frac{h_4 x^\alpha}{Q^2 (1-x)} + \frac{h_6 x^{2\alpha}}{Q^4 (1-x)^2} \right]$$

yields the following two-representative - solutions of equal statistical significance

$$\begin{aligned} h_6 &= 0, \quad h_4 < 0, \quad \alpha = 3.7 \quad , \\ h_6 &> h_4 > 0, \quad \alpha = 0.96 \quad . \end{aligned}$$

Unfortunately, no error analysis of these fits is available.

The fact that the  $x$  dependence of the twist-4 term is not yet resolved must be kept in mind when the relative magnitude of the twist-4 and twist-2 components is

considered. The data points themselves are too limited, both in number and accuracy, to make a reasonable fit when the exponents of  $x$  and  $(1-x)$  are also allowed to vary. The method of analysis of most experiments is to assume an  $x$ -dependence and then fit  $\Lambda$  and  $h_4$ . There are three experiments which have analyzed a combined sample of high energy data plus another experiment's low energy data. The results are:

BEBC + GARGAMELLE:  $Q^2 > 1.5 \text{ GeV}^2$ ,  $\mu_4(x) = x/(1-x)$

$$\left. \begin{array}{l} \Lambda < 0.44 \text{ GeV} \\ -.16 < h_4 < .6 \text{ GeV}^2 \end{array} \right\} 90\% \text{ C.L.}$$

CDHS + SLAC:

$Q^2 > 2.0 \text{ GeV}^2$ ,  $\mu_4(x) = x^\alpha/(1-x)$

$$\left. \begin{array}{l} .2 < \Lambda < .4 \text{ GeV} \\ -.7 < h_4 < 1.5 \text{ GeV}^2 \end{array} \right\} \text{with no restriction on } \alpha$$

EMC + SLAC:

$Q^2 > 5.0 \text{ GeV}^2$ ,  $\mu_4(x) = x^2/(1-x)^2$

$$\left. \begin{array}{l} \Lambda \approx 0.15 \text{ GeV} \\ h_4 \approx 0.6 \text{ GeV}^2 \end{array} \right\} \text{Best fit } \chi^2/\eta = 1.0$$

It is difficult to draw a conclusion from these results since they not only assume different  $\mu_4(x)$  but they also combine experiments with different systematics to obtain the large  $x$ - $Q^2$  domain required for such a fit. This combining of experiments with very different  $\langle Q^2 \rangle$  and targets can be quite misleading. Since the experiments often have targets of different  $Z$ , smearing corrections will be different and,

indeed, there are preliminary indications that the  $F_i$  are themselves  $z$  (!) dependent. Thus, fitting across boundaries of domains dominated by different experiments could influence not only the value of  $h_4$ , currently under discussion, but also the form of  $\mu_4(x)$  and the value of  $\Lambda$  when higher twist is ignored. The only completed experiment to date which does not share this difficulty is the SPS Gargamelle<sup>18</sup> experiment. The  $Q^2$  range of this experiment is  $0.5 < Q^2 < 50.0 \text{ GeV}^2$  and, assuming  $\mu_4(x) = x/(1-x)$ , the allowed values for  $\Lambda$  and  $h_4$  were

$$\left. \begin{array}{l} \Lambda < 0.27 \\ -.26 < h_4 < 0.0 \end{array} \right\} Q^2 > 0.5 \text{ GeV}^2 ,$$

$$\left. \begin{array}{l} \Lambda < 0.65 \\ -.5 < h_4 < .7 \end{array} \right\} Q^2 > 2.0 \text{ GeV}^2 .$$

The allowed contours (90% C.L.) are displayed in Fig. 7. If the BEBC results (90% C.L.), which are based on the same form of  $\mu_4(x)$ , are also included, then the domain becomes constrained to (Fig. 8)

$$\left. \begin{array}{ll} \Lambda < 0.32 & (h_4 > 0) \\ \Lambda < 0.44 & (h_4 > 0) \\ -.2 < h_4 < .5 \end{array} \right\} Q^2 > 2.0 \text{ GeV}^2 .$$

These are then the most stringent limits that can be placed on the twist-4 contribution at this time. There are strong

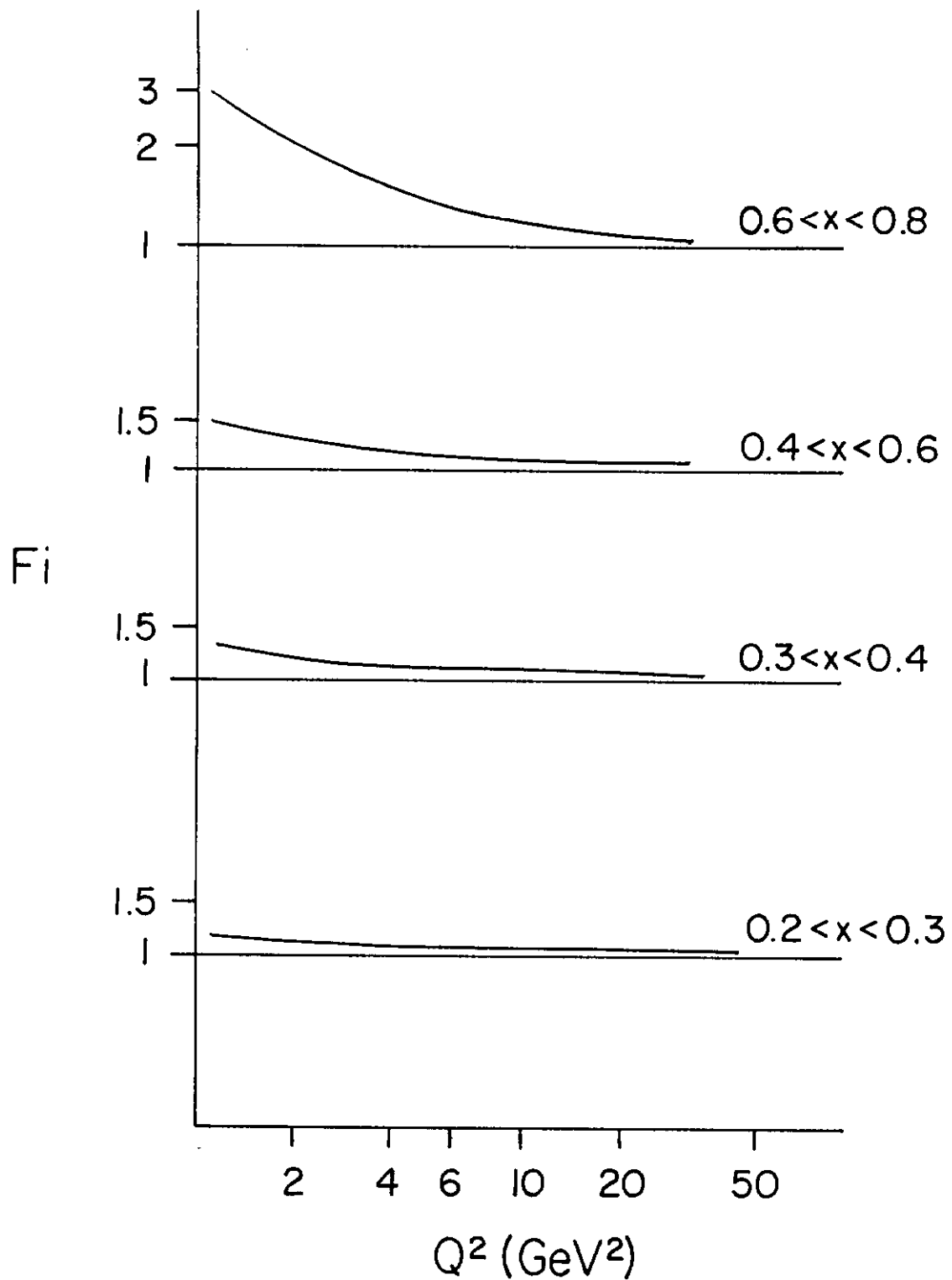
indications from the CDHS and SPS Gargamelle data that twist-4 alone (plus target mass corrections) cannot explain the observed  $Q^2$  behavior of the structure functions. This would imply that  $\Lambda=0$  and  $|h_4| \neq 0$  is not allowed and, indeed, the CDHS "favored" values suggest  $.2 < \Lambda < .4$  and  $-.2 < h_4 < .25$ . However, it is impossible to attach a statistical significance to these limits from the published analysis.

An obvious conclusion of this brief summary is that both the experimental and theoretical studies of higher twist phenomena are just beginning. The experimental difficulties are considerable. The ideal higher twist experiment would have minimal high  $x$  smearing (Fermi motion) corrections, a large sample of high  $x$  events and good hadron energy resolution down to 0 (1 GeV). This would suggest an enormous amount of running time using a hydrogen or deuterium target within a detector which, considering the required hadron energy resolution, has yet to be developed!

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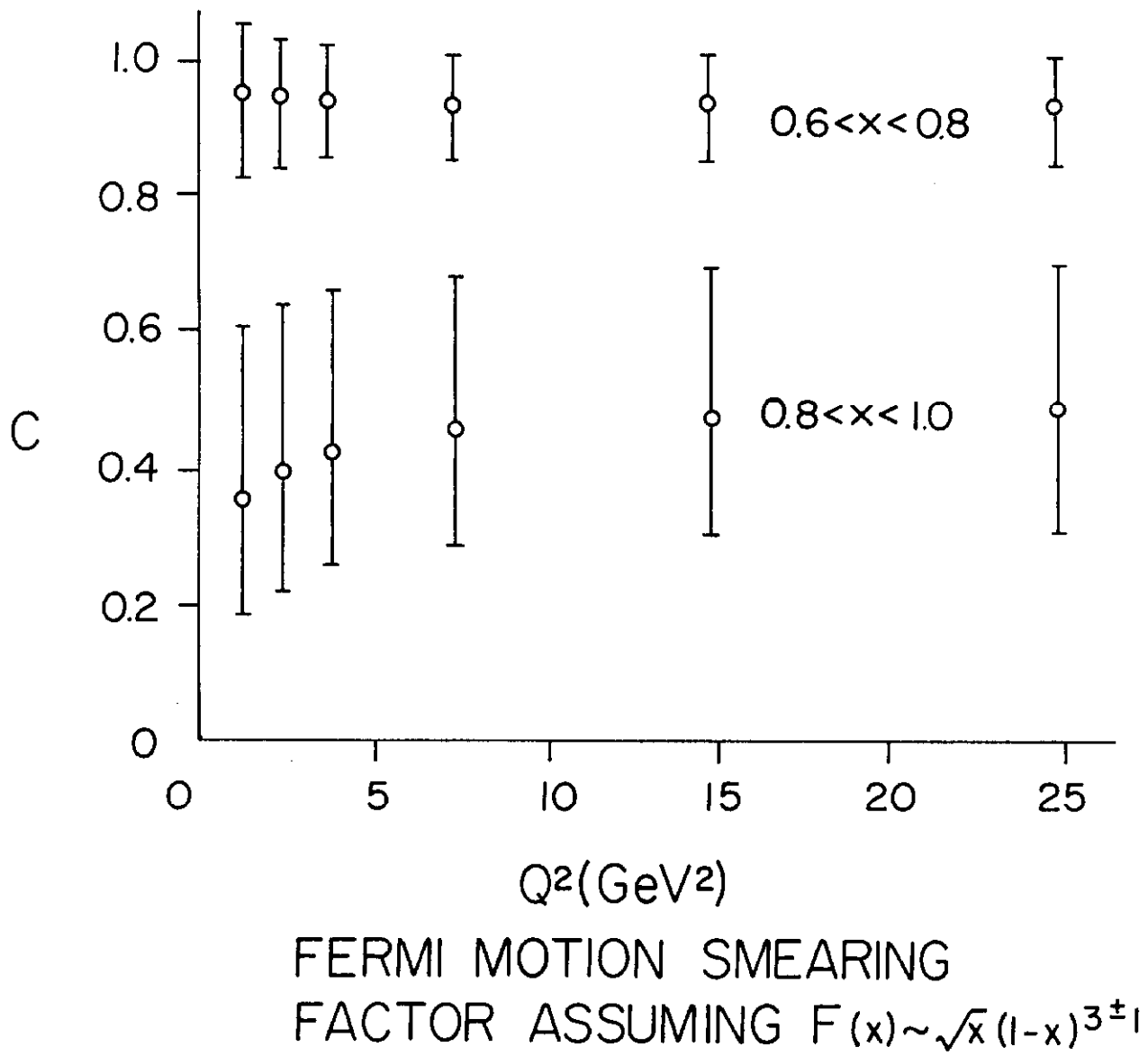
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FIG. 1

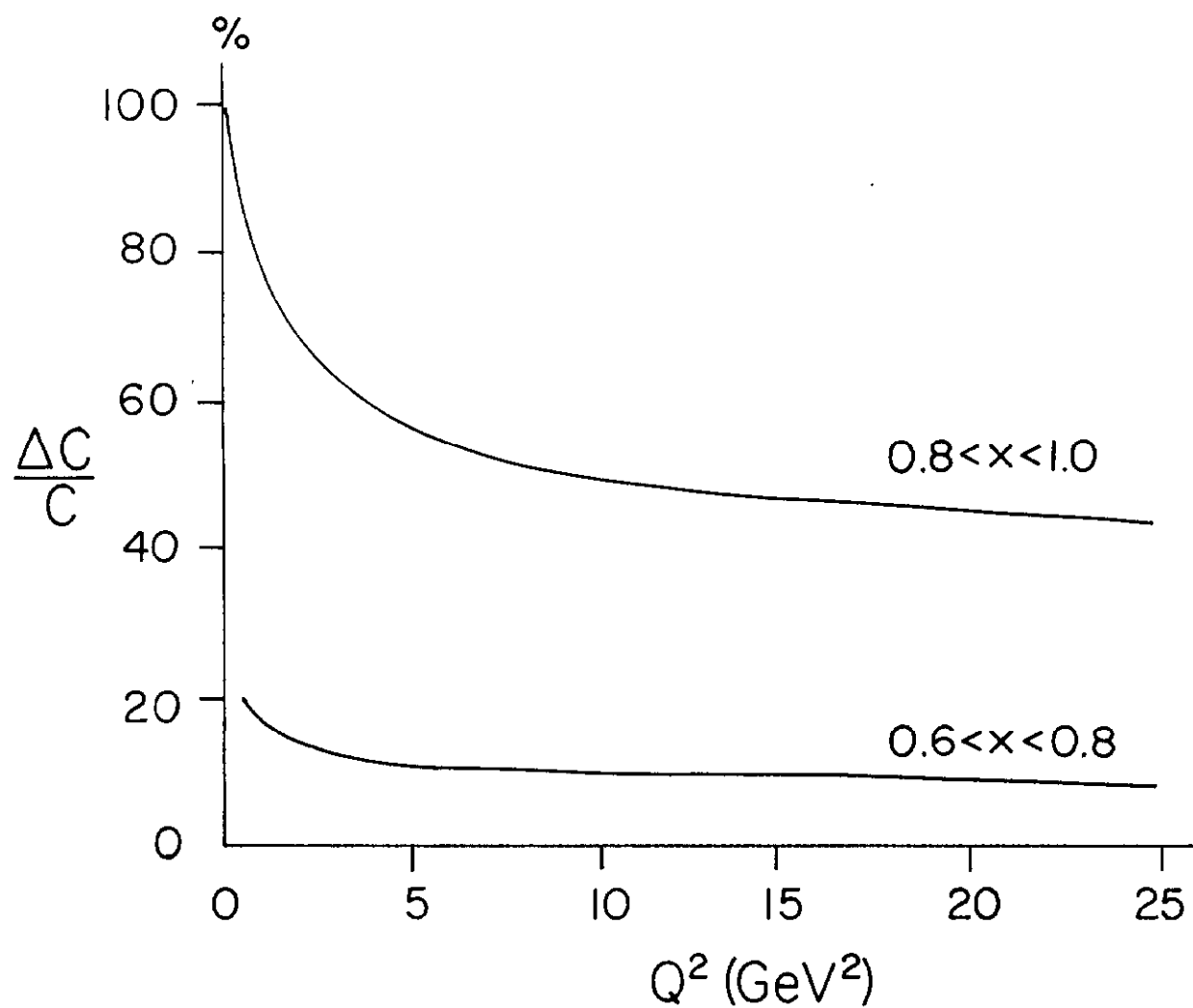


CORRECTING FOR TARGET MASS  
EFFECTS VIA RESCALING IN  $\xi$

FIG. 2a

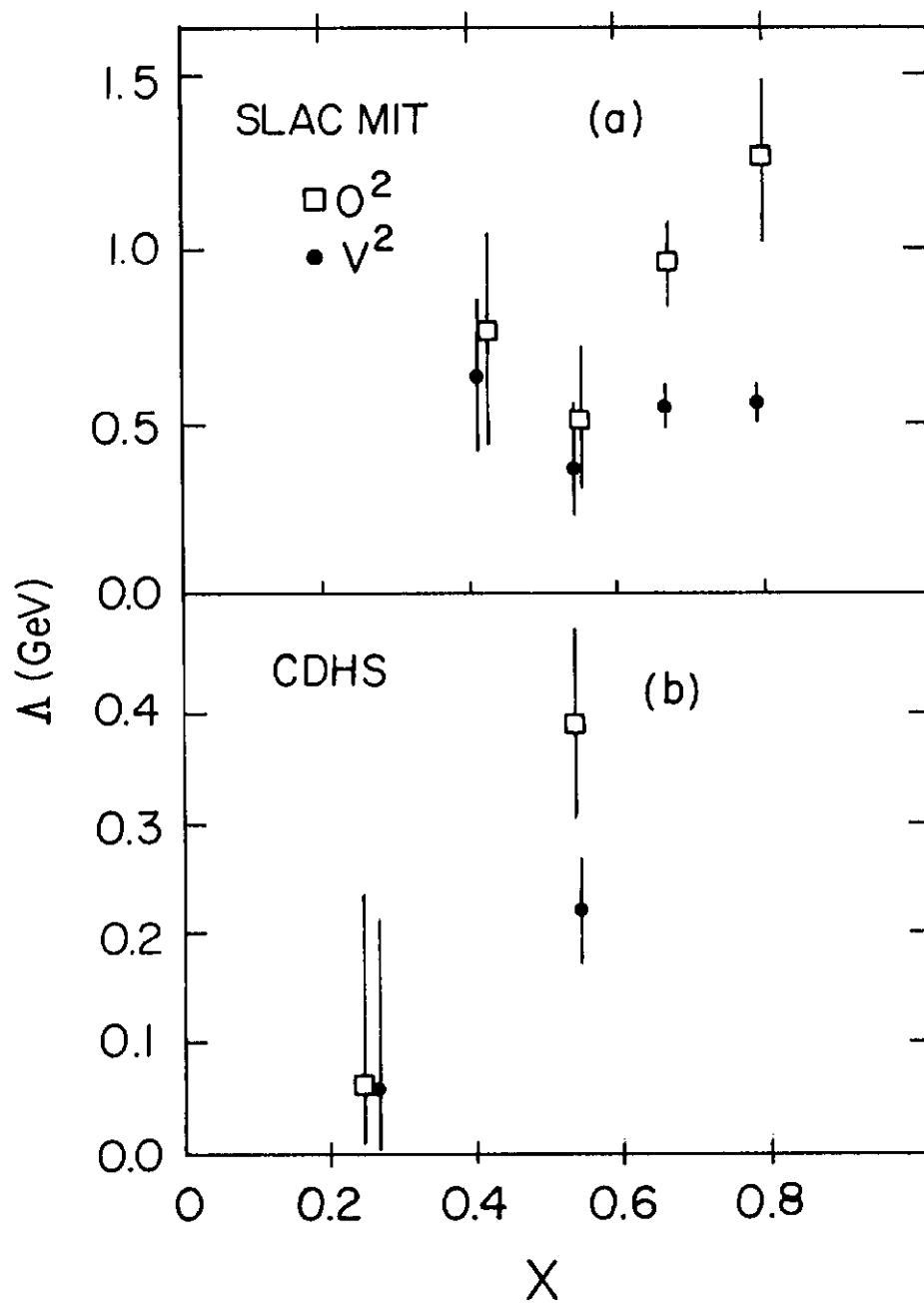




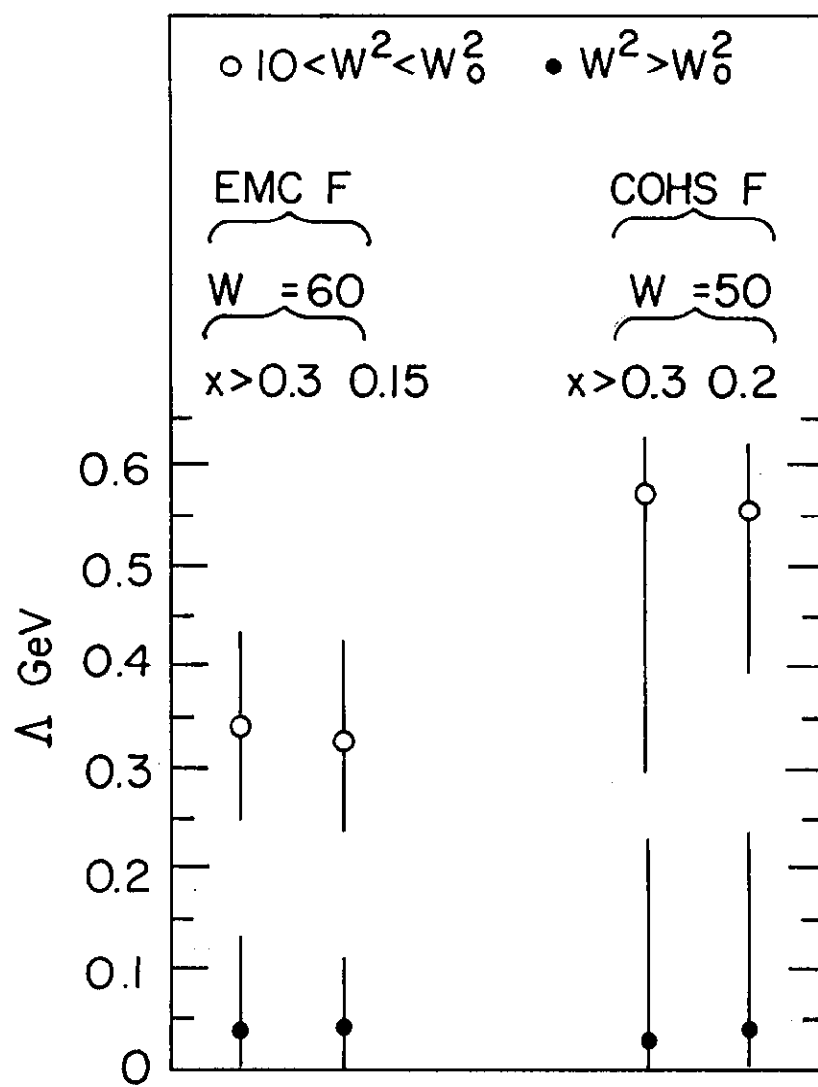


ACCURACY OF FERMI  
SMEARING FACTOR

FIG. 3a



$x$  DEPENDENCE OF  $\Delta$   
(From Ref. 4)



$W^2$  DEPENDENCE OF  $\Lambda$   
 (From Ref. 5)

FIG. 3c

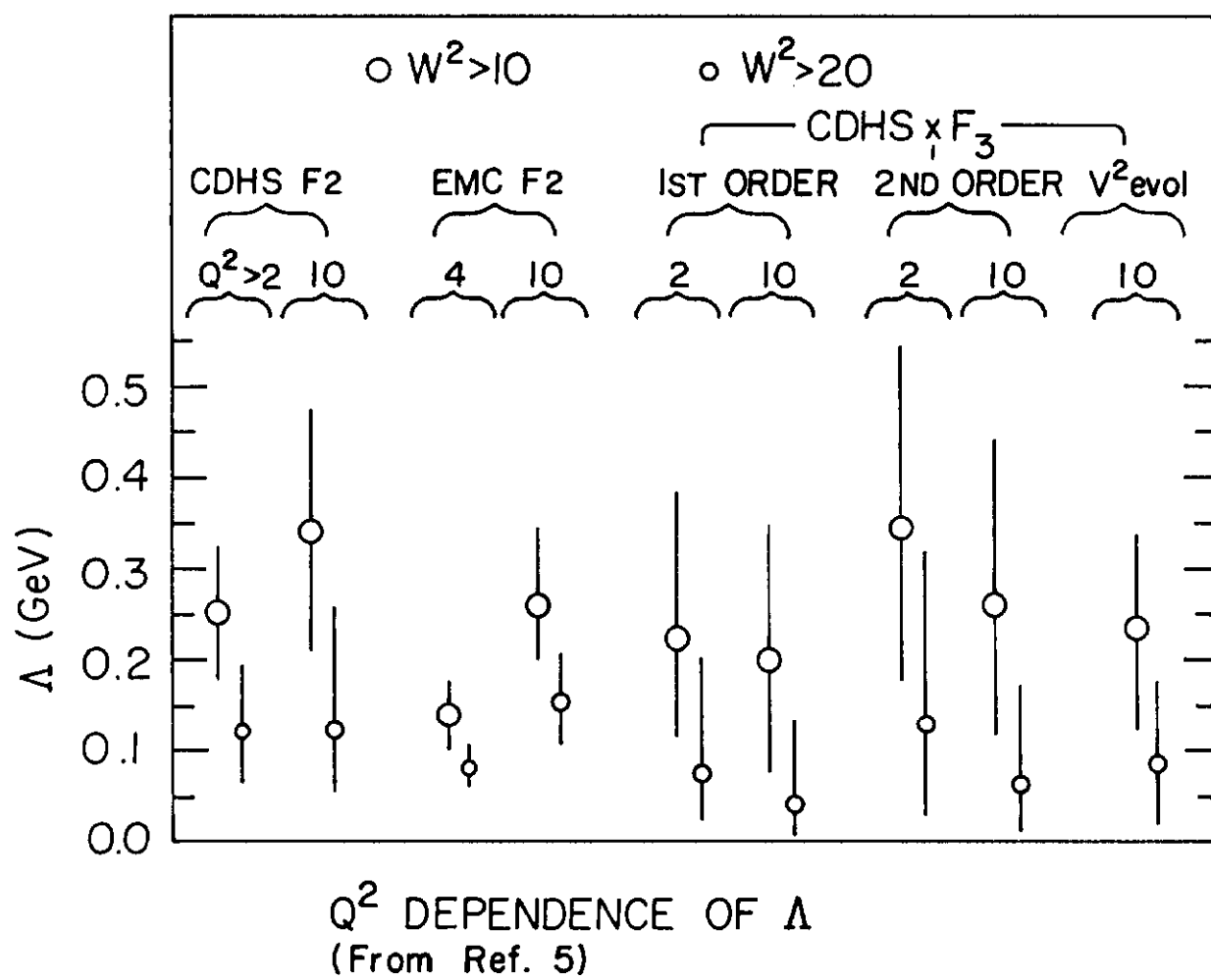
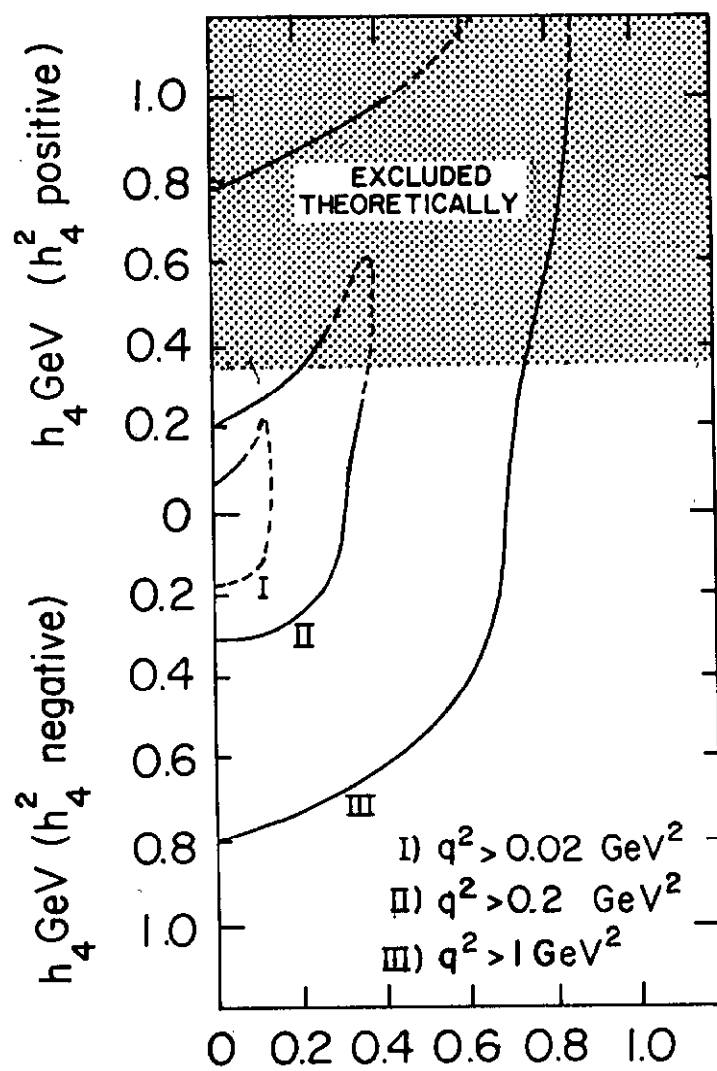


FIG. 4



ALLOWED REGIONS IN  
THE  $\Lambda H_4$  PLANE

FIG. 5

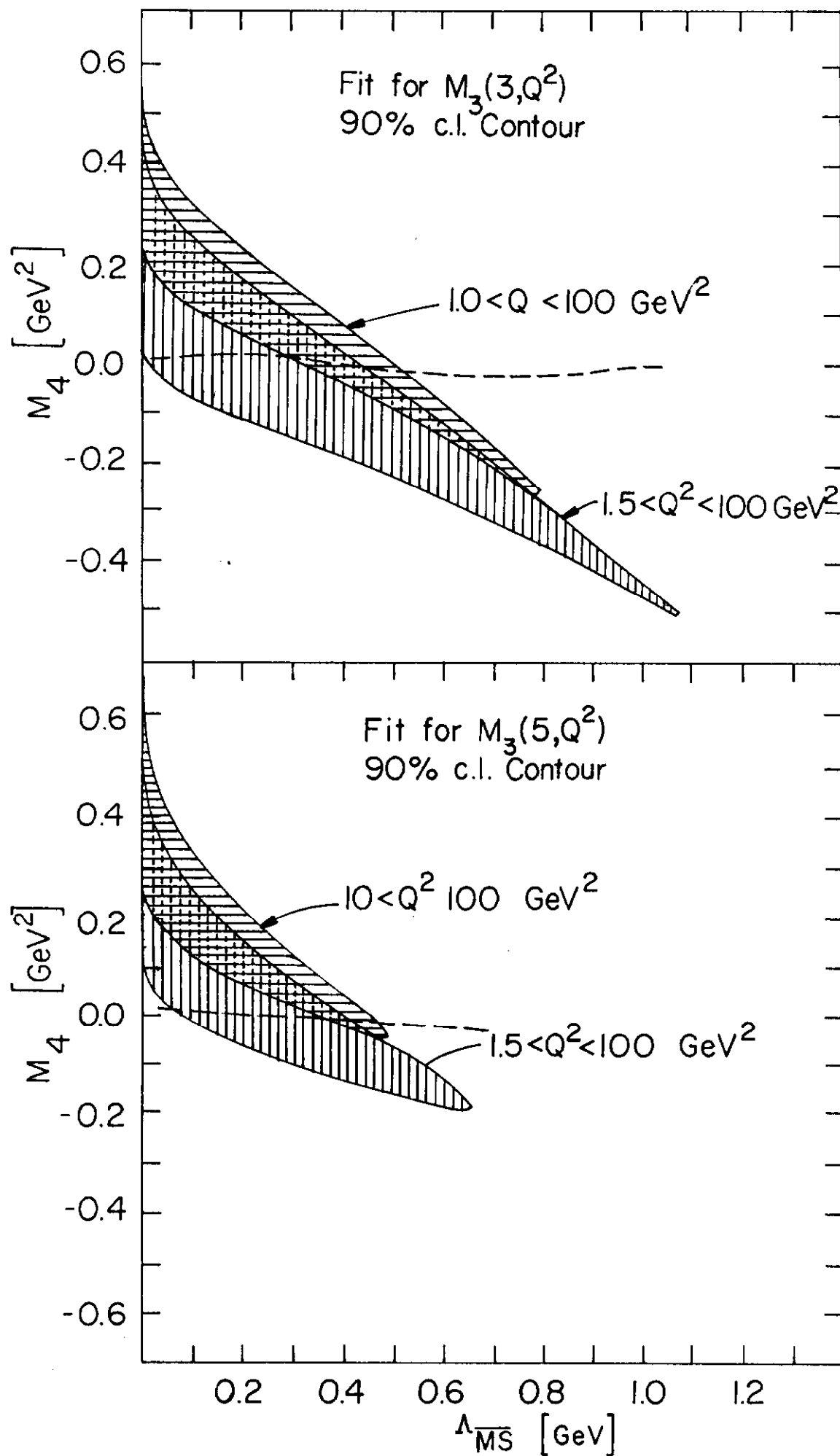


FIG. 6

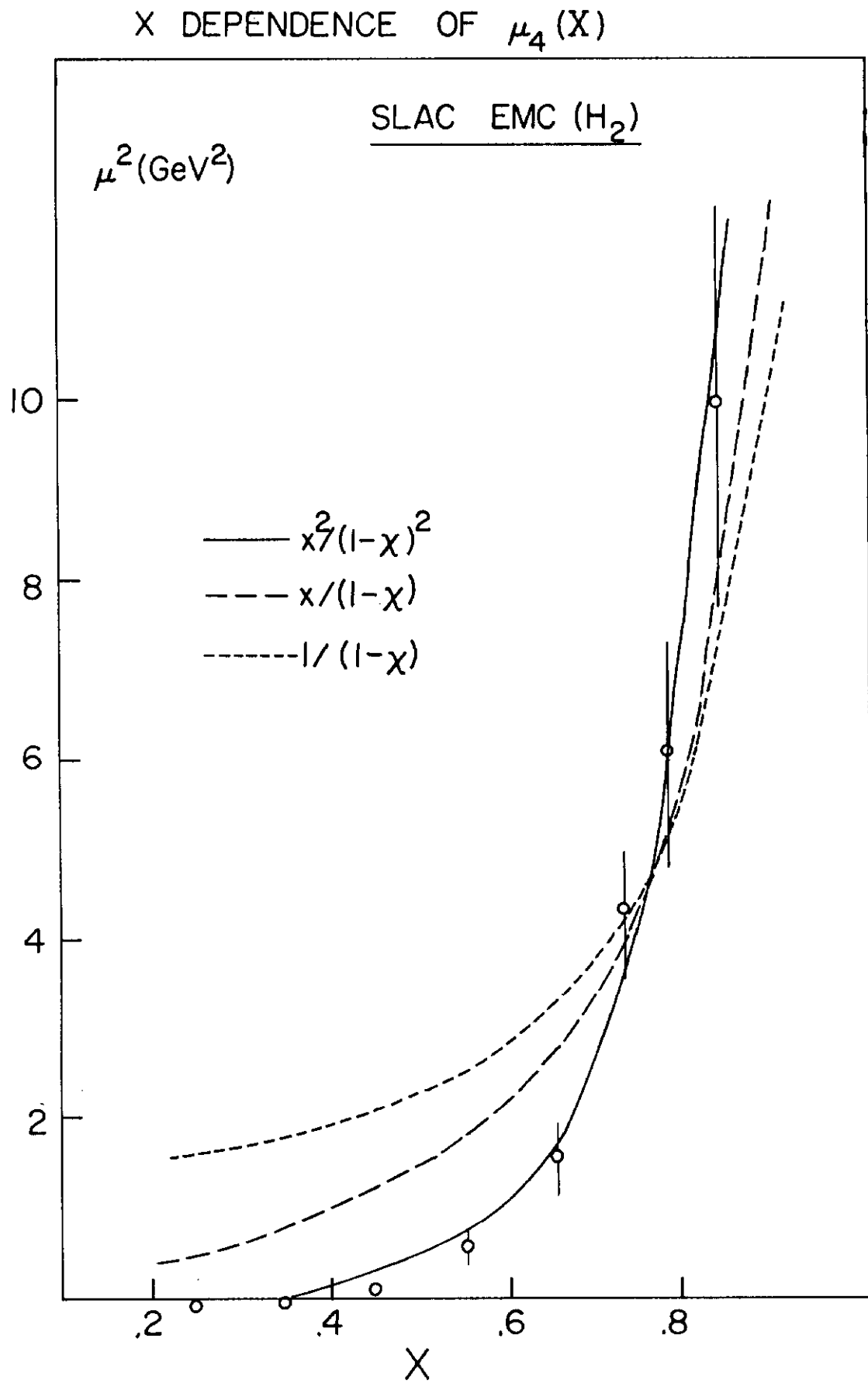


FIG. 7

Excluded (90% C.L.) GGM

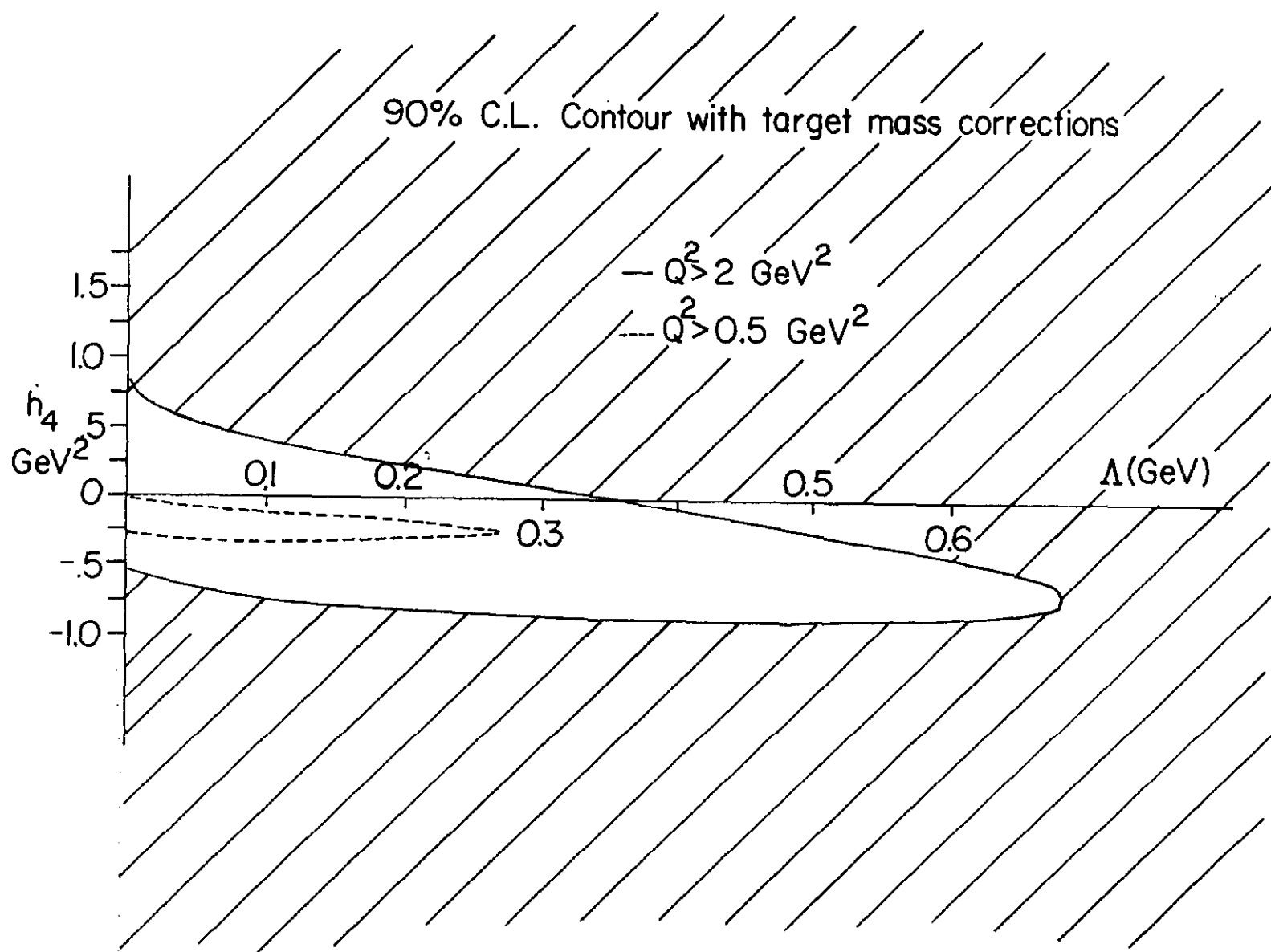




Fig. 8

